Chapter 10

Quasi-Geostrophic Implications

When you look at a weather chart with all its isolines and plotted data, you need a framework upon which to interpret what you see. Quasi-geostrophic theory provides one such framework. It allows you to imply specific things about what is happening within the atmosphere and provides a basis for expectations on how the atmosphere will change in the short-term. The purpose of this chapter is to explore some aspects of quasi-geostrophic (QG) theory. QG concepts have been the applied in operational forecasting for at least 50 years and are a primary part of the weather analysis process.

What is the QG Approximation?

The quasi-geostrophic approximation is used to produce a set of equations that apply to synoptic scale motion. This approximation includes the following assumptions:

a. There is no friction.
b. The horizontal winds can be approximated by geostrophic winds in the momentum equations.
c. The horizontal advection is approximated by geostrophic advection in the thermodynamic equation.
d. Vertical advection of momentum is neglected.
e. Static stability is based on a basic-state value with variations in the vertical only.

For a rigorous development of the quasi-geostrophic equations, you are referred to Bluestein (1992) or Holton (2004).

The equations developed using the quasi-geostrophic approximation can be applied to the atmosphere above the friction layer and away from mesoscale systems. This means that they can be used in the middle to upper troposphere (the 500-250 mb layer) to diagnose certain aspects of atmospheric motion. For example, the QG omega equation can be used to infer the location of synoptic scale vertical motion, while the QG tendency equation can show how pressure surfaces will change over time. Development of these equations involves expressing physical processes as terms in the equations.

The Omega Equation

A primary concern of forecasters is the formation of clouds and precipitation. Clouds are formed by one of two processes. The first process occurs when air near saturation rises, cools, saturates and condenses out water to form clouds. The second process involves the vertical mixing of warm, humid air with cool air. The omega equation can help identify areas where the first process is likely to occur on the synoptic scale, assuming sufficient moisture is available.

Three forms of the omega equation have been developed over the years: the classical omega equation; the Trenberth form; and the Q-vector form. We are not interested in the mathematical development of these equations, but want to apply the physical interpretation of these equations to our weather charts in order to identify areas of upward vertical motion.

Classical Omega Equation: This form of the omega equation has been around for decades. Vertical motion is proportional to two terms:

$$\underbrace{-f\ \partial/\partial p\ [-\mathbf{V_g}\cdot\nabla(\zeta + f)]}_{\text{Term A}} + \underbrace{h\ \nabla^2(-\mathbf{V_g}\cdot\nabla\theta)}_{\text{Term B}}$$

Term A represents the rate of increase with height of the advection of absolute geostrophic vorticity; Term B is the thickness advection.

If positive vorticity advection (PVA) is increasing with height, upward vertical motion is expected. If negative vorticity advection (NVA) is increasing with height, downward vertical motion is expected. Because vorticity advection typically increases with height from the mid-troposphere to jet stream levels, examination of the vorticity advection at 500 mb is often used to infer vertical motion. Strictly speaking, however, this approach should be used with caution.

Because thickness is proportional to the mean temperature of the layer which it represents, the second term if often interpreted as thermal advection. Specifically, warm air advection (WAA) is associated with upward vertical motion and cold air advection (CAA) is associated with downward vertical motion.

Trenberth Omega Equation: This form of the omega equation was developed in the late 1970s. Operational use of this equation was overshadowed by the Q-vector form of the equation that appeared in the early 1980s. Nevertheless, studies have shown that it can be useful in identifying areas of upward synoptic scale vertical motion. In this form vertical motion is proportional to:

$$-(\partial \mathbf{V}/\partial z)\cdot\nabla(\zeta + f)$$

that is, to the advection of absolute vorticity by the thermal wind. The thermal wind is the difference (vertical shear) in the geostrophic wind from the bottom to the top of the layer for which the thickness is calculated. Upward vertical motion is indicated by positive isothermal vorticity advection (PIVA) while downward vertical motion is associated with negative isothermal vorticity advection (NIVA). PIVA occurs when the thermal wind flows from higher values of vorticity to lower values of vorticity.

Experience has shown that if the 700-300 mb thickness is superimposed on the 500 mb vorticity pattern, the thermal wind can be inferred from the thickness pattern and used to advect the vorticity. Thermal wind is related to the thickness pattern in the same way that geostrophic wind is related to height contours on a constant pressure chart.

Q-Vector Form of the Omega Equation: This form of the omega equation is currently in vogue. The vertical motion is proportional to:

$$\nabla\cdot\mathbf{Q}$$

that is, the divergence of the Q-vector. The Q-vector is a mathematical expression that has no explicit physical meaning. However it is easily calculated from thermal and wind gradients and can be applied to a variety of situations to infer synoptic scale vertical motion. In many ways it is more flexible than the other two forms of the omega equation.

For example, for a simple trough, upward vertical motion is indicated downstream from the trough with maximum vertical motion near the inflection point of the flow. For flow across thermal gradients, warm air advection implies lift while cold air advection indicates sinking motion. Both of these applications can also be derived from the previous forms of the omega equation.

Q-vectors can also be applied to a jet streak. It produces the four-quadrant result that is typically associated with a maximum in wind speed along a straight jet stream in the upper troposphere: upward vertical motion in the left exit and right entrance regions; and downward vertical motion in the right exit and left entrance regions.

[Equations after Djurić (1994).]

Application of the Omega Equation

The operational application of the quasi-geostrophic equations uses the physical interpretations from the equation to infer the location of synoptic scale vertical motion. The following list summarizes these relationships:

- For troughs in the westerlies:
    - Lifting is located downstream from the trough, and upstream from the ridge.
    - Sinking is located upstream from the trough, and downstream from the ridge.
    - Maximum vertical motion is near the inflection point in the flow.
- For thermal advection:
    - Warm air advection (WAA) favors lifting.
    - Cold air advection (CAA) favors sinking.
- For a straight jet streak:
    - Lifting is favored in the left exit and right entrance regions.
    - Sinking is favored in the right exit and left entrance regions.

This means that you can examine a middle to upper tropospheric chart and infer the location of upward vertical motion and potential areas of clouds and precipitation (assuming moisture is available).

For example, in Figure 10-1, applying a quasi-geostrophic interpretation, the area between the trough over Indiana and the downstream ridge over Vermont/New Hampshire would be an area of upward synoptic vertical motion. This is a rather broad area and can often be refined by looking at the thermal advection at the 700 mb level. Figure 10-2 shows these data. The area of warm air advection is outlined in red and extends from central Pennsylvania to southern Maine. This area is under the 500 mb upward vertical motion area but limits the low level vertical motion to New England. Low level cold air advection is outlined in blue and extends from western Virginia to Indiana. Generally, there is good correlation between the areas of quasi-geostrophic vertical motion on the 700 mb chart and cloudy areas on corresponding satellite imagery.

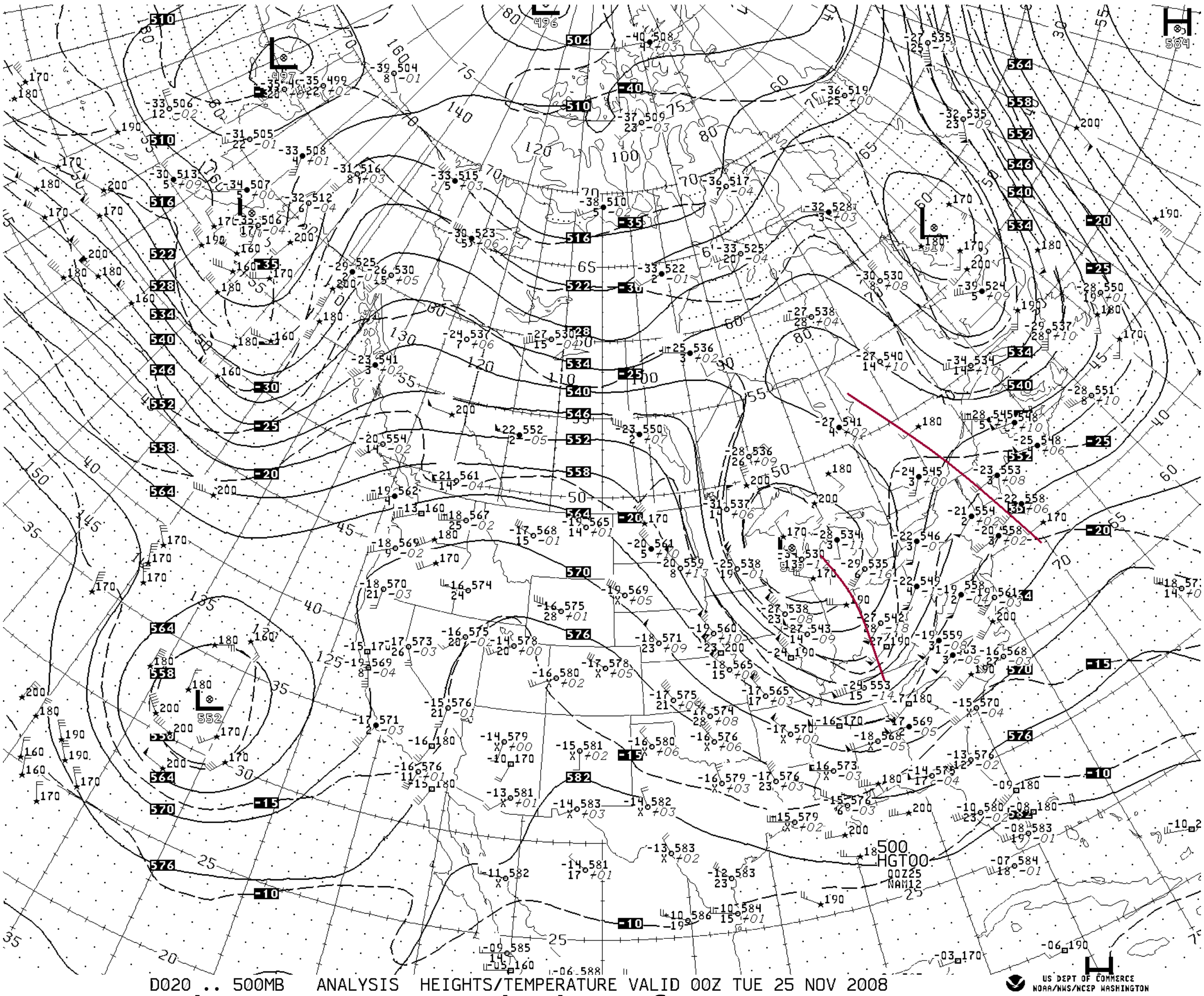

Figure 10-1: 500 mb Chart for 00UTC 25 MOV 2008

Operationally, thermal advection is frequently applied at the 850 mb level even though it is often in the boundary layer and may violate the quasi-geostrophic assumptions.

Similarly, the quasi-geostrophic equations do not apply to mesoscale weather systems and the concepts discussed above should not be applied at that scale.

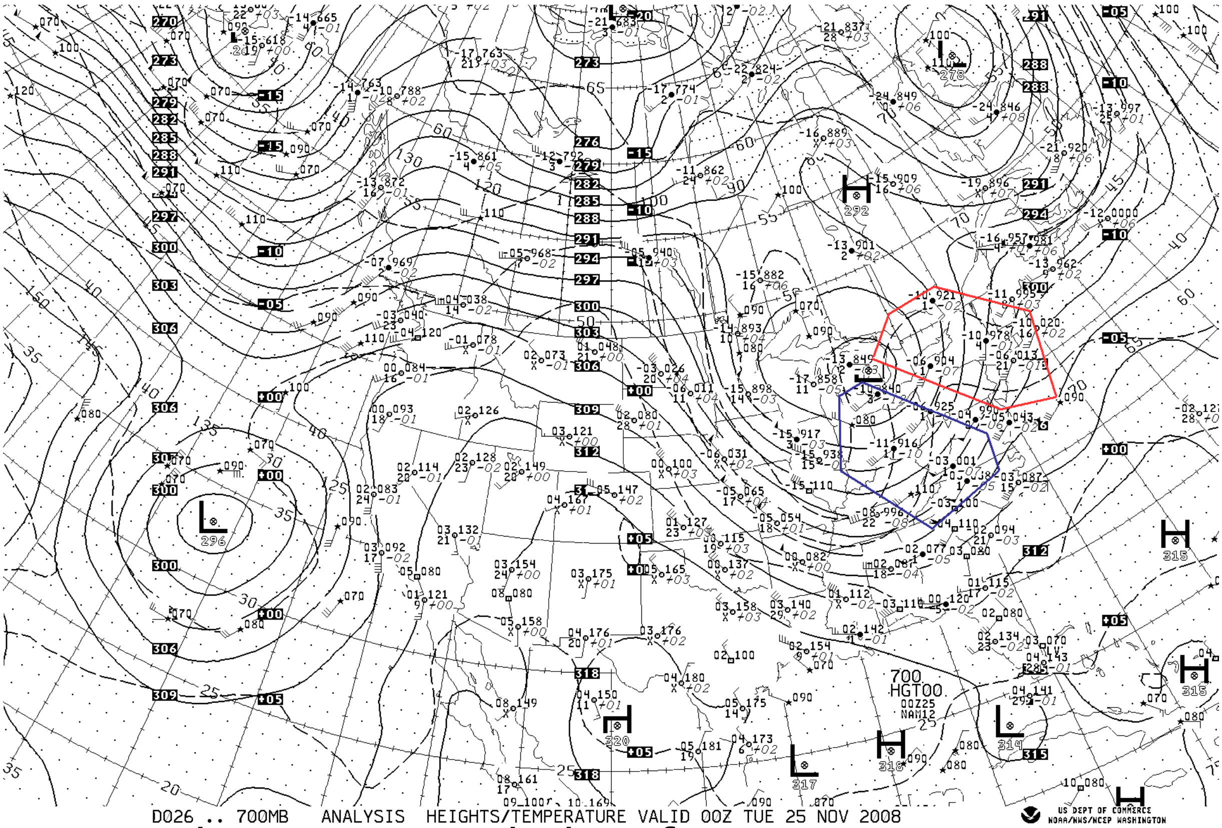

Figure 10-2: 700 mb Chart for 00UTC 25 NOV 2008

The Tendency Equation

Another equation that can be derived from quasi-geostrophic considerations is the tendency equation. This equation expresses the time tendency of the geopotential (Φ) of a pressure surface in the following terms:

$$\underbrace{-f_o V_g \cdot \nabla[(1/f_o)\nabla^2\Phi + f]}_{\text{Term A}} + \underbrace{(f_o^2/\sigma)\partial/\partial p[-V_g \cdot \nabla(\partial\Phi/\partial p)]}_{\text{Term B}}$$

Term A is the advection of the absolute vorticity by the geostrophic wind. This term controls the movement of troughs and ridges through the mid-latitude westerly flow. The advection of the relative geostrophic vorticity (first part of term A) tends to move the troughs and ridge eastward in the westerly flow. The advection of Earth vorticity (second part of term B) tends to move the troughs and ridges westward against the westerly flow. The combination of the two terms determines the final movement. This motion depends upon the wavelength of the trough or ridge.

Eastward movement of troughs and ridges dominates for shorter wavelength troughs, specifically, for wavelengths of approximately 3,000 km or less. Retrogression, or westward movement, prevails for wavelengths of approximately 10,000 km or more. For wavelengths between these two values, quasi-stationary or slow eastward movement is expected.

Term B indicates that the tendency of the geopotential field also depends upon the rate of change with height of the advection of the temperature by the geostrophic wind. Specifically, where cold advection decreases with height, the geopotential field will decrease; for warm advection, the geopotential field will increase. This implies that the thermal advection plays a significant part in the amplification of upper-level troughs and ridges for developing systems.

Application of the Tendency Equation

The application of the tendency equation has been replaced by computer forecast models in operational forecasting. These models show the movement of upper-level troughs and ridges, and the intensification and weakening of mid-latitude cyclones. Nevertheless, the equation, when applied to an upper-level chart, can still provide some short-term clues to trough and ridge movement and the potential for cyclone development.

For example, in Figure 10-1 the relative short wavelength associated with the troughs over the Gulf of Alaska, the Canadian prairie, and the northeastern U.S. imply a progressive flow based on Term A of the tendency equation. The warm and cold air advection seen in Figure 10-2 around the Great Lakes low center implies a potential for maintaining or intensifying a surface low associated with the mid-tropospheric low/trough.

Concluding Remarks

Quasi-geostrophic theory provides a theoretical basis for evaluating middle to upper tropospheric troughs and their impact on vertical motion, system movement, and intensification. Although limited to synoptic scale systems, quasi-geostrophic concepts have been used in operational forecasting for decades. They often provide clues to why things are happening within the atmosphere.

Draft: 6-18-2010
Final: 5-11-2011